THE ROLE OF THE LANDE *g*-FACTOR OF ELECTRONS IN EFFECTS OF INTEGER QUANTUM HALL'S PLATEAUX DISAPPEARING

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We indicate that a number of new effects can be observed in quantum electron structures under conditions of the integer quantum Hall effect. These are an effect of Hall's plateaux desappearing with even values of the Landau's levels filling factor and new effects of simultaneous disappearing of two and three Hall's plateaux. The phenomenon of intersection at the Fermi level of two or more spin-splitted Landau's levels is used as the basis of the analysis. The main results are demonstrated by the example of a single quantum well which is formed with two $GaAs/Al_{0.3}Ga_{0.7}As$ heterojunctions.

Introduction

In recent years, a great attention is paid to theoretical and experimental investigations of quantum electron systems (QES). One of the most famous and important phenomena in QES is the integer quantum Hall effect [1], which is characterized by a step-wise behaviour of quantum Hall's conductivity of two-dimensional electron systems with changing the magnetic field B under low temperatures. The value of steps is the quantum of conductivity e^2/h (here, e is the electron change, h is the Planck's constant).

The experimental observations of the integer quantum Hall effect in double quantum wells which are formed with $GaAs/Al_{0.3}Ga_{0.7}As$ heterojunctions leaded to the discovery of the effect of quantum Hall's plateaux disappearing with odd integer values of Landau's level filling factor N in 1990 [2]. The Hall's plateaux with $N = 3$ and $N = 1$ was discovered to disappear consequently with increasing the potential barrier between quantum wells. The origin of this effect was assumed in [2] to be fluctuations of the electron density in a strong magnetic field.

At the same time, the intersection of any pair spin-splitted Landau's levels at the Fermi level was considered in [2] as an alternative origin of quantum Hall's plateaux disappearing. However, as was shown in [2], in order to realize this intersection, the value of the magnetic field must be higher than that observed during the experiments.

In this paper, we indicate a possibility of observation of a number of new effects of integer quantum Hall's plateaux disappearing in QES with the help of the phenomenon of intersection at the Fermi level of two or more spin-splitted Landau's levels. The main results are illustrated by the example of a single rectangular quantum well which is formed with two $GaAs/Al_{0.3}Ga_{0.7}As$ heterojunctions. Let us note that, in the case of double quantum well, the influence of the effect of integer quantum Hall's plateaux disappearing on high frequency properties of such QES was studied in works [3 – 5].

Effects of Hall's Plateaux Disappearing in a Single Rectangular Quantum Well

Let us consider the effect of Hall's plateaux disappearing in a single quantum well of finite height which is formed with $GaAs/Al_{0.3}Ga_{0.7}As$ heterojunctions (see the inset in Fig. 1). Properties of the electronic system in a quantum well are considered in the framework of the grand canonical ensemble model ($\mu = \text{const}$, where μ is the Fermi energy). Following work [1], we assume that $\mu \approx 13$ meV and the height of the quantum well is $U_0 = 300$ meV.

As was obtained in [3 – 5], the energy of electrons E_{psq} and the Landau's level filling factor N are:

$$E_{psq} = E_p + \frac{\hbar\Omega}{2}(2s + 1 + qgm),$$

$$N = \sum_{p,s,q} n_F\left[\frac{E_{psq} - \mu}{k_B T}\right], \quad n_F(x) = (1 + e^x)^{-1}. \qquad (1)$$

Here, E_p is the energy of spatial quantization, $p = 1, 2, 3,...$ is the number of the energy level of spatial quantization, $s = 0, 1, 2,...$ is the number of Landau's level, $q = \pm 1/2$ is the spin quantum number (further, we will write the symbols $\downarrow$ and $\uparrow$ for each of the values $q = +1/2$ and $q = -1/2$, respectively), $\Omega = eB/m^* c$ is the cyclotronic frequency, m^* is the effective mass of electrons, $m = m^*/m_0$ is the relative mass of electrons, (m_0 is the mass of free electrons), g is the Lande g-factor of electrons, $n_F(x)$ is the Fermi function, $\hbar$ is the Planck constant, k_B is the Boltzmann constant, T is the temperature. Here, we limit ourselves to the case where the g-factor depends neither on the magnetic field B nor s (i.e., $g = \text{const}$).

It is evident from Eq. (1) that, in the case $k_B T \ll \hbar\Omega$, $k_B T \ll \mu$, the Landau's level filling factor N takes only integer values which equal to the number of energy levels lying under the Fermi level.

The energy levels of spatial quantization $E_p = \hbar^2 k_p^2 / 2m^*$ are known to be derived from the following expression for the energy spectrum of electrons in a single quantum well:

$$a \sin\left(\frac{\hbar k_p}{\sqrt{2m^* U_0}}\right) = \frac{p\pi - k_p a}{2}, \qquad (2)$$

where $p = 1, 2, 3, \ldots$

Initially, we will consider the dependence of the electron energy spectrum on magnetic field $E_{psq}(B)$ because the dependence $N(B)$ is determined by $E_{psq}(B)$.

The dependences $E_{psq}(B)$ are shown in Fig. 2 for $a = 646$ Å. The quantum numbers p, s, q of the corresponding energy level are indicated near each of the dependences. The Fermi level is indicated by the dash line. It should be noted that the spin orientations are shown for a positive g-factor value. In the case of negative g-factor values, these orientations should be changed by opposite ones.

The effect of Hall's plateaus disappearing can be explained by interaction of any two energy levels at the Fermi level. With increasing the magnetic field, these levels shift above the Fermi level and N decreases by two units.

The condition for intersection of two spin-splitted Landau's levels at the Fermi level looks

$$E_{p_1 s_1 q_1} = E_{p_2 s_2 q_2} = \mu. \qquad (3)$$

From this condition, we find the following expression for g_c and B_c:

$$g_c = \frac{(2s_1 + 1)(\mu - E_{p_2}) - (2s_2 + 1)(\mu - E_{p_1})}{m\,[q_2(\mu - E_{p_1}) - q_1(\mu - E_{p_2})]},$$

$$B_c = \frac{m}{\mu_B}\frac{q_2(\mu - E_{p_1}) - q_1(\mu - E_{p_2})}{q_2(2s_1 + 1) - q_1(2s_2 + 1)}, \qquad (4)$$

where $\mu_B = e\hbar / 2m_0 c$ is the Bohr's magneton.

The g-factor of electrons in QES is known to may differ by several ten times from its bulk value $g_b = 0.44$ and changes in the range from 0.34 to 19.3 [6]. Hence, we assume that the g-factor of electrons belongs to the range pointed. In addition, we consider the case of a wide enough quantum well with $a \geq 400$ Å. Such wide quantum wells were experimentally researched in [7].

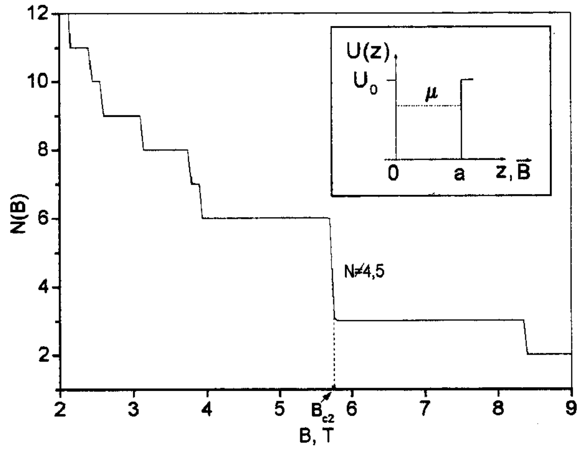

Fig. 1. Effect of simultaneous disappearing of two adjacent Hall's plateaux with $N = 4$ and $N = 5$ for $a = 646$ Å, $g_{c2} \approx 18.6$, and $B_{c2} = 5.7$ T. Inset shows a single rectangular quantum well

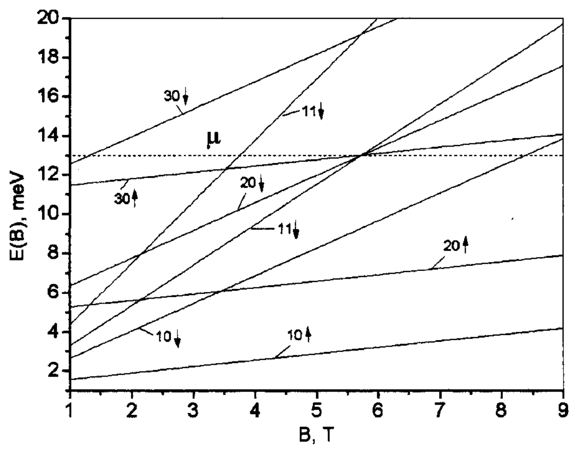

Fig. 2. Dependence of $E_{psq}(B)$ in a single rectangular quantum well for $a = 646$ Å and $g = 18.6$

Let us demonstrate that a set of new effects of Hall's plateaux disappearing can be observed in a single quantum well. These are the effect of simultaneous disappearing of two Hall's plateaux (with $N = 4$ and $N = 5$), effect of simultaneous disappearing of three Hall's plateaux (with $N = 2$, $N = 6$, and $N = 9$), and anomalous quantum Hall effect (it is characterized by a step-wise increase in Hall's conductivity with increase of the magnetic field). Henceforward, the critical values of the g-factor and the magnetic field corresponding to the effect of simultaneous disappearing of two (three) Hall's plateaus are denoted as g_{c2} and B_{c2} (g_{c3} and B_{c3}), respectively.

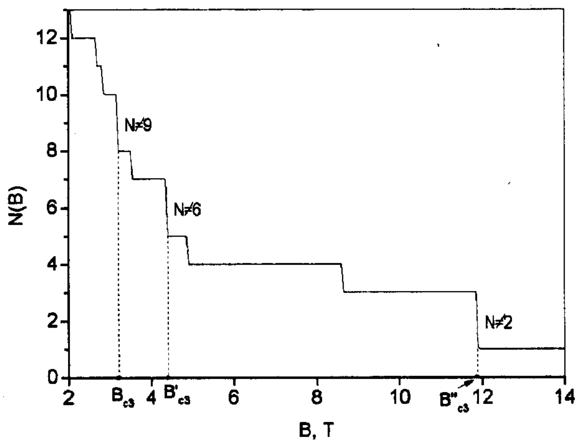

Fig. 3. Effect of simultaneous disappearing of three nonadjacent Hall's plateaux with $N = 2$, $N = 6$ and $N = 9$ for $g_{c3} \approx 4.76$ and $a = 689$ Å

Now we consider the effect of simultaneous disappearing of two Hall's plateaux on the dependence $N(B)$. The calculations show that, for $a \approx 646$ Å and $g_{c2} \approx 18.6$, the simultaneous intersection of three spin-splitted energy levels at the Fermi occurs. These are the energy levels $E_{11\uparrow}$, $E_{20\downarrow}$, and $E_{30\uparrow}$ (see Fig. 2). This intersection leads to the step down by three units in the dependence N.

The magnetic field at the point of the intersection is $B_{c2} = 5.7$ T. In the present case, the dependence of $N(B)$ is shown in Fig. 1. It is evident from this figure that the plateaux with $N = 4$ and $N = 5$ are absent on the dependence of $N(B)$.

Let us consider the effect of disappearing of three nonadjacent Hall's plateaux in the case of intersections the following pairs of energy levels at the Fermi level: ($E_{10\downarrow}$, $E_{20\uparrow}$), ($E_{11\downarrow}$, $E_{30\uparrow}$) and ($E_{21\downarrow}$, $E_{30\downarrow}$). As follows from Fig. 2, such intersections lead to the plateaux disappearing with $N = 2$, $N = 6$, and $N = 9$ on the dependence of $N(B)$ shown in Fig. 3. The critical value of the g-factor of electrons and the width of the quantum well are: $g_{c3} = 4.76$, $a = 689$ Å. The plateaux with $N = 9$, $N = 6$, and $N = 2$ disappear for $B_{c3} = 3.15$ T, $B'_{c3} = 4.36$ T, and $B''_{c3} = 11.89$ T, respectively.

Conclusion

In this work, a number of new effects has been predicted with the help of the phenomenon of intersections of two and more spin-splitted Landau's levels at the Fermi level. These are the effect of Hall's plateaux disappearing with even values of the filling factor N and the effect of simultaneous disappearing of two and three Hall's plateaux. They has been demonstrated by the example of the single rectangular quantum well which is formed with GaAs/ $Al_{0.3}Ga_{0.7}As$ heterojunctions. All these effects are conditioned by the discrete nature of the energy spectrum of electrons and, in principle, can be also observed in other quantum electron structures.

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