A NEW FIELD THEORY OF FUNDAMENTAL PARTICLES

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Proceeding from the dynamical structure of the space-time discontinuum (relativistic bi-Hamiltonian system) described in [1], a mathematically consequent theory of elementary particles and their interactions is built. As was shown earlier [11], this theory is free from ultraviolet divergences. In this paper, we find the evident form of field variables $f(x)$ and $\dot{f}(\dot{x})$ of the system and consider the quantum transitions $f \to \dot{f}$ in it. As a result, bilocal fields $\psi(X,Y)$ and non-point (smearing) particles related to them arise. The mass spectrum of 'bare" hadrons and leptons is found. Switching on interactions between fundamental particles is connected with the degeneration group of the ground state $\dot{f}$ of the system. The problem of reconstruction of the space-time continuum is considered.

1. Introduction

In the 20s, when the second quantization method was suggested, the serious mathematical inconsequent connected with impossibility to multiply improper functions was made. Its removal entailed some changes in our understanding of the nature of space-time and matter in small [1] and led to the consideration of dynamical structures of a new kind: the bi-Hamiltonian dynamical system which underlies the elementary particles and their interactions.

The main problem of the contemporary particle field theories based on the second quantization method is ultraviolet divergences (ultraviolet catastrophe). Physics met such a trouble already in the second time. In the first time, it arose in connection with the black-body radiation problem. The solution given by Planck was connected with the procedure of quantization of electromagnetic fields (accidentia) and led to the new, quantum theory. De Broglie and Schrödinger extended this principle to the matter (substantia) and postulated the existence of the fields $\psi(X)$ corresponding to the each kind of particles. In the non-formal physical language, the quantization of substantia and accidentia means to dismember them into indivisible entities which are called corpuscule and quanta, endowed by wave properties and called elementary particles. The quantum version of interacting fields given by Dirac, Heisenberg, and Pauli met again with the ultraviolet catastrophe. Taking into account the wave properties of matter (following Schrödinger, we say that both substantia and accidentia are joined in one and the same category of matter on the quantum level) allows one to lower a degree of divergences but not to remove them: it turns out only the matter quantization procedure is insufficient. Proceeding from the universality of this procedure, we have to subject to the quantization the third (and last) category of physics: the space-time (at present, one considers that there are only three categories of human knowledge: matter=substancia, space-time, and information=interaction=accidentia).

However, what does space quantization mean? Under this procedure, Snyder [13], for example, understood the formal transition from the space coordinates X as numbers to the operators $\hat{X}$. Now one speaks about non-commutative geometry [14]. Meanwhile, this approach did not lead to the main principle of constructing the particle theory yet.

Our approach is quite another. We consider that the ultraviolet catastrophe and inconsistency in the second quantization method are connected with the illegal extrapolation of the Newtonian concept of space-time and connected with it the Faraday – Maxwell field concept into the region of very small distances. Of course, the coordinates X are not the space itself (they are only a framework, the coordinate system or arithmetic net, which we draft on the space and on which none of the physical things depends – the relativity principle!). A non-formal meaning of the quantization procedure demands in this case to dismember the space into indivisible entities, i.e., into points (Euclid). In the Newtonian (classical) approach, the space is considered as a connected manifold (continuum) endowed by an additional differential structure. In this approach, points have no wave properties, the latter are leveled by the connectness (the same takes place in the case of matter). Therefore, we consider that in order to quantize the space, it is necessary, first of all, to break the connections between the points which made the space be a connected manifold and to make the space to be a non-connected manifold, i.e., a discontinuum. So, the space quantization is, first of all, a topological process (it needs to consider the strongest, discrete topology on the space), but not only it.

The analysis of the physical situation occurring at very small distances shows [1], that the space discontinuum actually aries in a new quantum mechanics which is a definite limit case of the Schrödinger wave mechanics. It is connected with the class of almost periodic functions in the momentum space and well adopted to the description of particle constituents – granules. The quantum version of granules (quantized field theory on the discontinuum) leads to a new kind of dynamical systems by which the inner dynamical structure of granules and their wave properties are described. Admitting the liberty of speech, we say that they are the waves surrounding an isolated point of the discontinuum.

Here, by proceeding from the dynamical structure of the discontinuum [1], we try to reconstruct the field theory of particles. We solve the equations describing the dynamics of our bi-Hamiltonian system and find the explicit form of its field variables f and $\dot{f}$ (Part 2). Their properties (including statistical) define all properties of fundamental particle fields and their interactions. In formation of the last, the quantum transition $f \to \dot{f}$ in the system plays a decisive role (Part 3). We can solve some important problems of particle theory: 1) to find the mass spectrum of fundamental particles (hadrons and leptons), 2) to discover the exis-tence of two branches of hadrons (baryons and mesons, Part 4), 3) to get the bilocal structure of particle fields and, hence, to find a way of solving the ultraviolet divergence problem (Part 5), 4) to connect the problem of switching on the particle interaction with degeneration of the ground state $\dot{f}$ of the bi-Hamiltonian system, and 5) to describe the mechanism of formation of the space-time continuum (Part 6).

In this paper, the simplest model of a bi-Hamiltonian system based on the Heisenberg algebra $h_8^{(*)}$ is considered. In this model connected with the Hamilton quaternions, the Lorentz symmetry is represented by the group $\mathrm{GL}(2,\mathbf{C})$, but the isotopic symmetry is represented by the $U(1)$ group (in which there is no electric charge). Therefore, this model is not realistic (all fundamental particles – in this model-barions, mesons, and leptons – are uncharged). The appearance of the necessary isotopic symmetry $U(2)$ is connected with the $h_{16}^{(*)}$ algebra and with the transition from the Hamilton algebra to the Cayley algebra (see [2]). All symmetry properties of the relativistic bi-Hamiltonian system $U[h_8^{(*)}]$ (it is the envelope algebra over $h_8^{(*)}$) are connected with its states $f^{\Sigma}(x)$ and $\dot{f}_z(\dot{x})$. We begin our consideration by finding the explicit form of the latter.

2. States $f(x)$ and $\dot{f}(\dot{x})$. Ensembles f and $\dot{f}$

It was shown in [1] that the self-adjoint (Fock) representation of the algebra $h_8^{(*)}$ and, unitary representation, connected with it, of its automorphism group $\mathrm{Sp}^{(*)}(4,\mathbf{C})$ (dynamical group of the system) is not suitable for description of the bi-Hamiltonian system. In [3], the new non-Fock representation of $h_8^{(*)}$ and non-unitary representation, connected with it, of $\mathrm{Sp}^{(*)}(4,\mathbf{C})$ are built. Such a representation, is realized in the dual pair of topological vector spaces $\overrightarrow{F}, \overleftarrow{F}$ connected one with another by a non-Hermitian form $\langle \bullet , \bullet \rangle$. The latter is well adopted to the description of irreversible quantum transitions. This representations is interlaced with the extended Fock representation of the same algebra $h_8^{(*)}$ (see [3]). In the latter, the generators of $h_8^{(*)}$ are written down in the form of the Fock representation as

$$\Phi = \begin{pmatrix} \varphi_\alpha \\ \partial / \partial \bar{\varphi}_\alpha \end{pmatrix}, \quad \bar{\Phi} = \left(- \frac{\partial}{\partial \varphi_\alpha}, \bar{\varphi}_\alpha \right), \quad \alpha = 1, 2, \qquad (1)$$

where φ_α, $\bar{\varphi}_\alpha$ are the coordinates on the Lagrangian plane $L \subset h_8^{(*)}$ (they are spinors). But the carrier space is $\overrightarrow{F} = F_{\mathrm{F}} \otimes F_0$, where F_{F} is the Fock representation space of functions $f(\varphi_\alpha, \bar{\varphi}_\beta)$ and F_0 is the space of functions $f(\varphi, \bar{\varphi})$, where $\varphi \doteq \varphi_2$, $\bar{\varphi} \doteq \bar{\varphi}_2$ are supplementary variables (Lorentzian scalars). In this representation, 4-momenta of the bi-Hamiltonian system are written as

$$\rho_\mu = \bar{\varphi} \overset{+}{\sigma}_\mu \varphi, \quad \dot{p}_\mu = - \frac{\partial}{\partial \varphi} \bar{\sigma}_\mu \frac{\partial}{\partial \bar{\varphi}}, \qquad (2)$$

where $\overset{\pm}{\sigma}_\mu = (\vec{\sigma}, \pm i)$ are the Pauli matrices ($i\sigma_\mu$ are the Hamiltonian quaternions; the system in which all three fundamental constants $c = h = k = 1$ is used).

The state vectors $f(x)$ and $\dot{f}(\dot{x})$ of the system are functions (fields) on the group $T_{3,1}$ and $\dot{T}_{3,1}$, generated by the p_μ and $\dot{p}_\mu$, correspondingly. As $p^2 = \dot{p}^2 = 0$ (p_μ and $\dot{p}_\mu$ are isotropic 4-vectors, see (2)), the fields $f(x)$, $\dot{f}(\dot{x})$ describe the objects with zero 'mass" (remember that they are non-Lagrangian fields, because they exist only in the fiber [1]; we recall that the Lagrangian fields exist in the space-time continuum). We call them as quanta f and $\dot{f}$ [1].

Obviously, the solutions of the equations for f and $\dot{f}$ (see [1])

$$- i \frac{\partial}{\partial x_\mu} f(x) = p_\mu f(x), \quad - i \frac{\partial}{\partial \dot{x}_\mu} \dot{f}(\dot{x}) = \dot{p}_\mu \dot{f}(\dot{x}) \qquad (3)$$

may be written down in the form of $f(x) = e^{ipx} f_0$ and $\dot{f}(\dot{x}) = e^{i\dot{p}\dot{x}} \dot{f}_0$.

a) The states $\dot{f}_0$ are determined as the solutions of the stationary equations

$$\dot{p}_\mu \dot{f}_0 = \rho_\mu^{(0)} \dot{f}_0 \qquad (4)$$

which belong to the space F_0. Limited at $|\varphi| \to \infty$, they are written as

$$\dot{f}_0 = \dot{f}_z = Ce^{\bar{z}\varphi - \bar{\varphi} z} = Ce^{2i\,\mathrm{Im}\,\bar{z}\varphi}, \qquad (5)$$

where C is a normalized constant and z is a complex parameter. The eigenvalue $\rho_\mu^{(0)}$ is written as $\rho_\mu^{(0)} = \bar{z}\bar{\sigma}_\mu z = |z|^2 (0,0,-1,-i)$, where spinor $z = \begin{pmatrix} 0 \\ z \end{pmatrix}$. This implies that $\rho^{(0)^2} = 0$ and $\rho_0^{(0)} \le 0$, so that $\rho_\mu^{(0)} \in N_-$ (lower 'flap" of the light cone). So far as $\varphi, \bar{\varphi}$ are Lorentzian scalars, $\dot{f}_z$ are Lorentzian scalars too. Such states are strongly degenerated on the energy-momentum ρ_μ.

In fact, if we act on (4) by a transformation $T(\upsilon), \upsilon \in \mathrm{SL}(2\,\mathbf{C})$, we come to the equations $\dot{p}_\mu \dot{f}_z = \rho_\mu \dot{f}_\mu$, where $\rho_\mu = (L^{-1}(\upsilon))_{\mu\nu} \rho_\nu^{(0)} \in N_-$ (υ is an arbitrary transformation). As the cone vertex $\rho_\mu = 0$ (the parameter $z = 0$) is invariant under $L(\upsilon)$, the manifold of all ρ_μ is split into two submanifolds: open N_- and closed $\{0\}$ ones. Solution (5) may be written as

$$\dot{f}_z = Ce^{i\sqrt{-P^2}\,\sin\nu}, \qquad (6)$$

where $-P^2 = 2\pi_\mu \rho_\mu$ and $\nu = \arg\varphi - \arg z$. In fact, as $\pi_\mu = \overset{+}{\bar{\varphi}} \sigma_\mu \varphi$, $\pi\rho = \overset{+}{\bar{\varphi}} \sigma_\mu \varphi \bar{z} \bar{\sigma}_\mu z = 2|\bar{z}\varphi|^2$, where $\varphi \doteq \varphi_2$.

[1] In the fiber, the entities f and $\dot{f}$ accomplish a so-called 'vertical" motion with light velocity c. They may not move along the base (in the 'horizontal direction"; about terminology, see any course of fibration spaces), because the latter is a discontinuum. It is interesting to note that 4-momenta p_μ and $\dot{p}_\mu$ are the relativistic generalization of the potential and kinetic energies of an oscillator existing at a point of the discontinuum.

The Lorentz-invariant (normalized) measure on N_- is

$$d\mu_{\dot{f}} = \frac{2}{\pi} \theta(-\rho_0)\, \delta(\rho^2)\, d^4\rho \qquad (7)$$

and, on $\{0\}$, is

$$d\mu_0 = \delta^4(\rho)\, d^4\rho. \qquad (8)$$

We call the $d\mu_{\dot{f}}$ and $d\mu_0$ the measures of final states $\dot{f}_z$.

b) As $\pi_\mu = \overset{+}{\bar{\varphi}} \sigma_\mu \varphi$, we have $\pi^2 = 0$ and $\pi_0 = \bar{\varphi}_\alpha \varphi_\alpha > 0$ (see (2)). So that $\pi \in N_+$ (the upper arena of the light cone). Thus, the entities (quanta f) are characterized only by positive energies π_0, hence, they have no anti-entities. Therefore, they may not be ' second-quantized" (f's are pure classical fields). The Lorentz-invariant measure on the N_+ is connected with the measure on the Lagrangian plane L, $\prod_{\alpha=1,2} \frac{i}{4} d\varphi_\alpha \wedge d\bar{\varphi}_\alpha$ ($\wedge$ is the Cartan multiplication), and may be written through π_μ in the form

$$d\mu_f = \frac{1}{(2\pi)^{3/2}} \theta(\pi_0)\, \delta(\pi^2)\, d^4\pi \frac{d\nu}{2\pi}. \qquad (9)$$

It is the (normalized) measure on N_+ that is called the measure of initial states f.

It is worth noting that the stationary equation $p_\mu f_0 = \pi_\mu f_0$ is the identity, from which the states f_0 may not be definite. This tells that the initial states are shapeless: they are formed in the quantum transition $f \to \dot{f}$ only, which takes place in the bi-Hamiltonian system. The operator $\hat{M}^2 = 2\dot{p}_\mu p_\mu$ and its stationary group $G_{\hat{M}^2} = \mathrm{GL}_l(2,\mathbf{C}) \otimes U_i(1) \otimes H_i(1)$ play an important role in this formation. Here, $\mathrm{GL}_l(2,\mathbf{C}) = \mathrm{SL}_l(2,\mathbf{C}) \otimes U_l(1) \otimes H_l(1)$.

In the case of the extended Fock representation including the supplementary variables $\varphi, \bar{\varphi}$, the splitting of the phase transformations $U(1)$ and $H(1)$ into the Lorentzian $U_l(1), H_l(1)$, and isotropic $U_i(1), H_l(1)$, takes place [3].

The states f_0 are defined as eigenfunctions of the operator $\hat{M}^2$:

$$\hat{M}^2 f_0^\Sigma = F_\Sigma^0 f_0^\Sigma. \qquad (10)$$

They form a multiplet Σ of the group $G_{\hat{M}^2}$ (the latter plays the role of the degeneration group of the 'Hamiltonian" $\hat{M}^2$). Here, Σ is a set of quantum numbers $\Sigma = (s, F, D; Y, N)$ which define a finite-dimensional irreducible representation of the group $G_{\hat{M}^2}$ (here, s is the Dirac spin connected with the subgroup $\mathrm{SL}_l(2, \mathbf{C})$, F and D are the fermion charge and dilatation connected with the subgroup $U_l(1)$ and $H_l(1)$, Y and N are the hypercharge and isotonic quantum number connected with the subgroups $U_i(1)$ and $H_i(1)$). Thus, in the model of $h_8^{(*)}$, the isotopic symmetry is represented by the group $U_i(1)$. The states f_0^{Σ} are written down in the form of homogeneous polynomials $f_0^{\Sigma} = O^{\Sigma}(\psi_\alpha, \overline{\varphi}_\beta; \varphi, \overline{\varphi})$ of degree N in all the variables $\varphi_\alpha, \overline{\varphi}_\beta, \varphi, \overline{\varphi}$ and degree D in only variables $\varphi_\alpha, \overline{\varphi}_\beta$. Then Y is the difference between the numbers of variables φ_α, φ and $\overline{\varphi}_\beta, \overline{\varphi}$, and F is the difference between of φ_α and $\overline{\varphi}_\beta$. The difference $Y - F = S$ is called strangeness (in connection with this, we call the supplementary variables $\varphi, \overline{\varphi}$ as quanta of the strangeness; if $\overline{\varphi}$ sticks to the normal spinor φ_α, the strange spinor $\varphi_\alpha \overline{\varphi}$ arises; compare with the Heisenberg spurion formalism [4]). On the states O^{Σ}, the operator $\hat{M}^2$ has the eigenvalue

$$F_{\Sigma}^{0} = -[(N+4)^2 - Y^2] \tag{11}$$

(see Appendix). We call O^{Σ} as the particle 'skeleton". All internal symmetry properties of particles (the set quantum numbers Σ) are defined by the symmetry properties of skeletons. When the irreversible quantum transition $f \to \dot{f}$ takes place, the skeletons overgrow by mass and the dependence of the coordinates $x, \dot{x}$ (or X, Y, see below).

c) f_0^{Σ} is the state of an isolated (single) quantum f. In the case where quanta f are in the ensemble, it is necessary to take into account their statistical properties described by the Gibbs distribution function

$$w_f = \exp\left(-\frac{\overline{\varphi}\varphi}{T_f}\right), \tag{12}$$

where $\overline{\varphi}\varphi$ is the energy of f quanta (in the Lorentz system connected with the subspace F_0) and T_f is the temperature of quanta f.

In the early stage, the phase transition 'particle $\to$ granules" is reversible. In this case, quanta f are in the thermal equilibrium with particles (at a temperature T_f) like photons in the black body. Such an ensemble is characterized by the minimum of the free energy F_f, i.e., $\left(\frac{\partial F_f}{\partial N_f}\right)_{T_f, V_f} = 0$, (where N_f is the number of quanta f and V_f is the volume in which they are). This condition leads, as is known [5], to the <u>zero chemical potential</u> $\mu_f = 0$ of the ensemble f. We may consider also the ensemble f as a certain gas. As at every fiber (point), there is no more than one copy of the bi-Hamiltonian system and the number of fibers (points of the discontinuum), in which there are such copies, is countable (in the Universe, this number is even finite), but the whole number of fibers (points of the discontinuum), onto which the space is split in the phase transition 'continuum$\to$ discontinuum", is uncountable, the gas f is perfect, i.e., $\overline{n}_f \ll 1$ (the most of points is empty). Therefore, $e^{-\varepsilon_f/T_f}$ is the Boltzmann distribution function n_f at $\mu_f = 0$ (see [5]). In the strong degeneration case, n_f is the Fermi and Bose distribution functions depending on the statistics of skeletons O^{Σ}. Taking into account the statistical properties of quanta f, we may write $f^{\Sigma}(x) = w_f f_0^{\Sigma}(x)$, where $f_0^{\Sigma}(x) = e^{i\pi x} f_0^{\Sigma}$. In the quantum transition $f \to \dot{f}$ which will be considered further, the Gibbs function w_f may be written in the relativistic Juttner's form (see [6]):

$$w_f = \exp\left(-\frac{\pi\rho}{2\mu^2}\right) = \exp\left(-\frac{-P^2}{4\mu^2}\right), \tag{13}$$

where $-P^2 = 2\pi\rho \geq 0$ and $\mu^2 = 3T_f T_{\dot{f}}$, where $T_{\dot{f}} = \frac{1}{3}|z|^2$ is the temperature of the ensemble of quanta $\dot{f}$ (see further). In fact, as $\pi\rho$ is a relativistic invariant, and we have $\rho_\mu = \rho_\mu^{(0)}$ and $\varphi_\alpha = \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$ in the special Lorentz system connected with the subspace F_0, so $\pi\rho = 2|z|^2|\varphi|^2$ in this system. From here, we get $\overline{\varphi}\varphi = \frac{\pi\rho}{2|z|^2} = \frac{\pi\rho}{6T_{\dot{f}}}$ and therefore $\frac{\overline{\varphi}\varphi}{T_f} = \frac{\pi\rho}{2\mu^2} = \frac{-P^2}{4\mu^2}$ [2]. Note that 4-momentum ρ_μ of quanta $\dot{f}$ plays the role of 4-velocity in (13). Note also that w_f is a homogeneous function on the group $T_{3,1}$ and does not depend on the 'angle momentum" $\pi_\mu x_\nu - \pi_\nu x_\mu$ and coordinates x_μ. As a result, the field of quanta f being in the ensemble is written in the form

$$f^{\Sigma}(x) = w_f f_0^{\Sigma}(x) = e^{i\pi x} e^{-\frac{-P^2}{4\mu^2}} O^{\Sigma}(phi). \tag{14}$$

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[2]The parameters T_f and $T_{\dot{f}}$ in the model $h_{16}^{(*)}$ were calculated in [3]. Now we do not pay attention to the essential difference between statistical reasonings in the frames of usual (unitary) and non-unitary quantum schemes (compare with [12]).

Statistical properties of the ensemble of quanta $\dot{f}$, in which quanta f undergo the quantum transition $f \to \dot{f}$ (see further), may be taken into account in a quite analogous manner. They are described by the distribution function

$$w_{\dot{f}} = \exp\left(-\frac{\rho_0 + \mu_{\dot{f}}}{T_{\dot{f}}}\right) \approx \exp\left(-\frac{\mu_{\dot{f}}}{T_{\dot{f}}}\right) = \frac{1}{Z}, \qquad (15)$$

where $\mu_{\dot{f}}$ is the chemical potential of the ensemble $\dot{f}$. It is non-zero because the quanta $\dot{f}$ are accumulated due to transitions $f \to \dot{f}$. In (15), $T_{\dot{f}}$ is a temperature of $\dot{f}$ introduced above. In (15), we consider that $\rho_0 << \mu_{\dot{f}}$. As a result, the field of quanta $\dot{f}$ being in the ensemble is written in the form

$$\dot{f}_z(\dot{x}) = \frac{C}{Z} e^{i\rho\dot{x}} e^{i\sqrt{-P^2}\sin\nu}. \qquad (16)$$

We emphasize that the fields $f^{\Sigma}(x)$ (especially $\dot{f}_z(\dot{x})$) exist only at an isolated point of the discontinuum (at the fiber). In the space-time continuum, they are not. It is a peculiar confinement of states of the bi-Hamiltonian system (quanta f and $\dot{f}$).

3. Transition Amplitudes

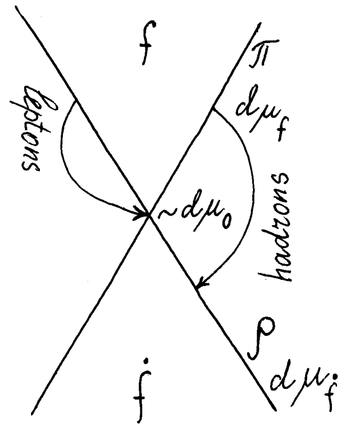
We have already seen that the relativistic bi-Hamiltonian system is a kind of two-level dynamical system. As a result of the phase transition 'particles→ granules“ (or 'Langrangian system → bi-Hamiltonian system“), the latter is on the upper level f. Under the condition of a superstrong compression of quanta f, when the size of the cloud $f(x)$ is $\sim 10^{-20}$, m (see [1]), the irreversible quantum transition $f \to \dot{f}$ takes place (the form $\langle \dot{f}, f \rangle$ is non-Hermitian, see below). The transition is described by the matrix element or transition amplitude $O^{\Sigma}(x, \dot{x})$ which is a result of integration of the product $\dot{f}_z(\dot{x}) f^{\Sigma}(x)$ with the measures of initial and final states $d\mu_f$ and $d\mu_{\dot{f},0}$. If we denote the transition matrix element as

$$\langle \dot{f}_z(\dot{x}), f^{\Sigma}(x) \rangle = \int \overline{\dot{f}_z(\dot{x})} f^{\Sigma}(x)\, d\mu_f, \qquad (17)$$

the transition amplitude $O^{\Sigma}(x, \dot{x})$ is

$$O^{\Sigma}(x, \dot{x}) = \int d\mu_{\dot{f},0} \langle \dot{f}_z(\dot{x}), f^{\Sigma}(x) \rangle =$$

$$= \langle\langle \dot{f}_z(\dot{x}), f^{\Sigma}(x) \rangle\rangle. \qquad (18)$$



We denote still $X = \frac{1}{2}(x + \dot{x})$, $Y = \frac{1}{2}(x - \dot{x})$, then $O^{\Sigma}(x, \dot{x})$ will be denoted as $O^{\Sigma}(X, Y)$. We can see that there are two topologically different transitions: transitions from the upper area of the light cone (measure $d\mu_f$) to its lower area (measure $d\mu_{\dot{f}}$), called hadronic transition, and transitions from the upper area into its vertex (measure $d\mu_0$) called leptonic one (see Figure).

In the first of them, heavy particles arise (hadrons), in the second, the light particles (leptons) arise (we recall that, in the model $h_8^{(*)}$, all particles will be uncharged). Here, we consider only hadronic transitions.

If to denote $P = \pi - \rho$, $Q = \pi + \rho$, transition amplitudes may be written as

$$O^{\Sigma}(X, Y) = \frac{C}{(2\pi)^{3/2}} \int e^{iPx + iQY}\, \theta(P_0 + Q_0) \times$$

$$\times\, \theta(P_0 - Q_0)\, \delta(P^2 + Q^2)\, \delta(PQ)\, O^{\Sigma}(P, Q)\, d^4P \frac{d^4Q}{2\pi}, \qquad (19)$$

where

$$O^{\Sigma}(P, Q) = \frac{1}{Z} \int e^{-i\sqrt{-P^2}\sin\nu - \frac{-P^2}{4\mu^2}} O^{\Sigma}(\varphi) \frac{d\nu}{2\pi} \qquad (20)$$

(these magnitudes will be calculated latter). If to act on amplitude (19) by the D'Alembertian operator $\Box_X = \frac{\partial^2}{\partial \mathbf{X}^2} - \frac{\partial^2}{\partial X_0^2}$ and to take into account that $p^2 = \dot{p}^2 = 0$, we get

$$\Box_X O^{\Sigma}(X, Y) = \langle\langle \dot{f}_z(X - Y), \hat{M}^2 f^{\Sigma}(X + Y) \rangle\rangle, \qquad (21)$$

where $\hat{M}^2 = 2\dot{p}p$ is the above-introduced operator. Denote $[\hat{M}^2, e^{ipx}]\, e^{-ipx} = \hat{\mathcal{J}}(x)$. We have $\hat{\mathcal{J}}(0) = 0$. This condition allows us to separate the homogeneous part in the equation, which we get now. Let us introduce the notation

$$J^{\Sigma}(X,Y) = \langle\langle \dot{f}_z(X - Y), \hat{\mathcal{J}}(X + Y)\, f^{\Sigma}(X + Y)\rangle\rangle . \quad (22)$$

It is not difficult to show that

$$\hat{M}^2 f^{\Sigma} = F_{\Sigma}(-P^2)\, f^{\Sigma}, \quad (23)$$

where $f^{\Sigma} = w_f O^{\Sigma}(\varphi)$, and

$$F_{\Sigma}(-P^2) = F^0_{\Sigma} + (N + 5)\frac{-P^2}{\mu^2} - \frac{(-P^2)^2}{4\mu^4} \quad (24)$$

(see Appendix), where F^0_{Σ} is (11). In the accepted notations, Eq. (21) can be written as the inhomogeneous equation

$$(\Box_x - F_{\Sigma}(\Box_X))\, O^{\Sigma}(X,Y) = J^{\Sigma}(X,Y) \quad (25)$$

(here, $J^{\Sigma}(X,Y)$ plays the role of a source for the amplitude $O^{\Sigma}(X,Y)$). This equation may be represented as (the left side of (25) is a polynomial of the second degree in $\Box_X$)

$$D(\Box_X)\, O^{\Sigma}(X,Y) = (\Box_X - M^2_{\Sigma B}) \times$$

$$\times (\Box_X - M^2_{\Sigma M})\, O^{\Sigma}(X,Y) = 4\mu^4 J^{\Sigma}(X,Y), \quad (26)$$

where

$$M^2_{\Sigma B} = 2\mu^2\{N + 5 - \mu^2 +$$

$$+ \sqrt{\mu^4 - 2\mu^2(N + 5) + Y^2 + 2N + 9}\,\} \quad (27)$$

and

$$M^2_{\Sigma B} = 2\mu^2\{N + 5 - \mu^2 -$$

$$- \sqrt{\mu^4 - 2\mu^2(N + 5) + Y^2 + 2N + 9}\,\}. \quad (28)$$

4. Mass Spectrum of Fundamental Particles

The mass spectrum of 'bare“ (non-interacting) fundamental hadrons is determined by formulas (27), (28) (we call particles arising in the quantum transition $f \to \dot{f}$ fundamental). The mass formula for such particles may be written as

$$M^2_{\Sigma} = \left(\frac{kh}{c}\right)^2 2\mu^2\{N + 5 - \mu^2 + (-1)^{F+1} \times$$

$$\times \sqrt{\mu^4 - 2\mu^2(N + 5) + Y^2 + N + 9}\,\}, \quad (29)$$

in which $F = 1$ and $F = 0$ correspond to the barionic (fermionic) and to the mesonic (bosonic) branches, correspondingly. Thus, in the suggested theory, the mass problem of fundamental particles is connected with the eigenvalue problem for the operator $\hat{M}^2$ on the states f^{Σ}. The existence of two branches, barionic and mesonic ones, is a consequence of taking into account the statistical properties of the ensemble of quanta f, i.e., the function $w_f \approx n_f$, which is split in the limit of strong degeneration (this limit is not realized for fundamental hadrons, see further) into two distribution functions: fermionic n_f^F ($F = 1$) and bosonic n_f^B ($F = 0$) ones described by the formula

$$n_f = \frac{1}{\exp\left(\frac{\pi\rho}{2\mu^2}\right) + (-1)^{F+1}} \quad (30)$$

(on the mass shell, we have $\pi\rho = -P^2/2 = M^2_{\Sigma}/2$). For large $\pi\rho/2\mu^2$ (that takes place for hadrons), both cases ($F = 1$ and $F = 0$) have the same Bolzmann limit (13). The factor $(-1)^{F+1}$ appearing in (29) is connected with (30) and follows from the more precise dispersion law corresponding to the states $f^{\Sigma} = n_f O^{\Sigma}(\varphi)$, where n_f is (30):

$$D_{\Sigma}(X) = (1 + (-1)^F n_f)(1 + 2(-1)^F n_f)X^2 +$$

$$+ 4\mu^2(\mu^2 - (N + 5)(1 + (-1)^F n_f))X +$$

$$+ 4\mu^4((N + 4)^2 - Y^2) = 0. \quad (31)$$

Here, $X = 2\pi\rho$ and n_f is (30) (formula (26) is obtained from (31) by the condition $n_f \ll 1$; the roots of Eq. (31) are denoted as before by M^2_{Σ}).

We can see that, by handling only with the limit of strong degeneration, we can identify roots (27), (28) with barionic and mesonic branches (by the reason of $M^2_{\Sigma}/4\mu^2 \gg 1$, this limit is not reached for hadrons, but for leptons and baroleptons only, see below).

The theorem about the connection of the roots of Eq. (31) with fermionic charge F is a consequence of the following fact: at any temperature T, the mean energy $\overline{\pi_0^{(F)}}$ of the fermionic skeleton O^{Σ} is larger than the energy $\overline{\pi_0^{(B)}}$ of the bosonic skeleton O^{Σ} (for the same quantum numbers N and Y). The relation $\overline{\pi_0^{(F)}}/\overline{\pi_0^{(B)}}$ is a monotonic function of the temperature T, which is equal to ∞ when $T = 0$ and to 9/8 when $T = \infty$ [5]. In our case, $T = T_f \ll 1$, see [3].

Leptons arise when the bi-Hamiltonian system transits from the upper area of the cone into its vertex. In this case, 4-momentum $\rho_\mu = 0$ (so that the parameters z and μ^2 are equal to zero) and therefore $w_f = 1$ (in this limit, the function n_f behaves in the well-known manner [5]). In this case, it follows from (18) that the amplitudes of lepton transitions $O^\Sigma(X,Y)$ coincide with the matrix elements $\langle 1/Z, e^{i\pi(X+Y)}O^\Sigma \rangle$ and satisfy the equation $\Box^2_X O^\Sigma(X,Y) = 0$ (see (26)). They represent the fields of particles with zero mass. Thus, the masses of 'bare" leptons are sero: $M_\Sigma = 0$(in formula(29), the parameter $\mu^2 = 0$ corresponds to leptons). As a consequence, the lepton spectrum is more poor than the hadron spectrum (the masses of charged leptons in the model $h^{(*)}_{16}$ are calculated in [7]).

It is important to consider another case where $\mu^2 \to \infty$. It has sense only for the fermionic branch ($F = 1$). It corresponds to baroleptons playing an important role in the new weak interaction theory [8]. In this limit, $M^2_\Sigma < 0$ (baroleptons are tachions in our space X), and $|\, M^2_\Sigma \,| >> 1$ (in this limit, meson masses $M_{\Sigma M} \to \infty$ and $n^B_f \to -1$, that has no sence). In this limit, transition amplitudes obey the equation $(\Box_X - F^0_\Sigma)\, O^\Sigma(X,Y) = J^\Sigma(X,Y)$.

5. Fields of Fundamental (Lagrangian) Particles

a) As is known, the solution of the inhomogeneous equation (26) can be written in the form $O^\Sigma(X,Y) = G^\Sigma(X,Y) + \psi^\Sigma(X,Y)$, where G^Σ is a partial solution of the inhomogeneous equation and ψ^Σ is a general solution of the homogeneous equation

$$(\Box_X - M^2_{\Sigma B})(\Box_X - M^2_{\Sigma M})\, \psi^\Sigma(X,Y) = 0. \qquad (32)$$

The solution of the latter have the known physical sense as they are written in the form

$$\psi^\Sigma(X,Y) = \frac{1}{(2\pi)^{3/2}} \int e^{iPX}\, \delta(P^2 + M^2_\Sigma)\, O^\Sigma(P,Y)\, d^4P \qquad (33)$$

and, in the asymptotic $|\,X\,| >> |\,Y\,|$, they are

$$\psi^\Sigma(X,0) = \psi^\Sigma(X) = \frac{1}{(2\pi)^{3/2}} \int e^{iPx}\, \theta(P_0)\, \psi^\Sigma(P)\, \delta(P^2 + M^2_\Sigma)\, d^4P, \qquad (34)$$

where $\theta(P_0)\,\psi^\Sigma(P) = O^\Sigma(P,0)$. Such solutions are used in the usual local particle field theory: they are usual Lagrangian fields. On the contrary, the magnitudes G^Σ are not known in the usual theory. Now we make use this circumstance in order to determine the explicit form of the function $O^\Sigma(P,Y)$ in (33). Namely, we demand that G^Σ be orthogonal to ψ^Σ in the sense of Stueckelberg's scalar product:

$$\int \psi^\Sigma(X,Y)\, \overline{G^\Sigma(X,Y')}\, d^4X = 0. \qquad (35)$$

It follows from this condition that

$$O^\Sigma(P,Y) = \int e^{iQY}\, \theta(P_0 + Q_0)\, \theta(P_0 - Q_0)\, \times$$

$$\times\, \delta(P^2 + Q^2)\, \delta(PQ)\, O^\Sigma(PQ)\, \frac{d^4Q}{2\pi} \qquad (36)$$

and the normalized constant C appeared in (5) must be equal to $C = \delta(0)$. It is infinite and, therefore, all that consist it (i.e., $f_z = (\dot{x})$, $O^\Sigma(X,Y)$, $J^\Sigma(X,Y)$, $G^\Sigma(X,Y)$) are infinite without immediate physical meaning. By the appearance of such objects, the suggested scheme differs from the standard one. It is very important to keep in mind that $\delta(0)$ is appeared here in non-observed magnitudes (in the standard scheme, $\delta(0)$ is appeared in observed magnitudes what is not admitted).

b) In the suggested theory, the bilocal fields $\psi^\Sigma(X,Y)$ correspond to non-point (smearing) objects. It follows from their explicit form (33), (36) that they may be written in the form of smearing local fields

$$\psi^\Sigma(X,Y) = F_\Sigma\left(Y; -i\frac{\partial}{\partial X}\right) \psi^\Sigma(X), \qquad (37)$$

where smearing operator F_Σ is written in the approximation not depending on the kind of particle as

$$F(Y,P) = \frac{1}{2\pi} \int e^{iQY}\, \delta(P^2 + Q^2)\, \delta(PQ)\, d^4Q. \qquad (38)$$

Here, P is a time-like 4-momentum and Q is a space-like one. A smearing particle is something of the alloy of a usual particle (variables P, X) and a tachion (variables Q, Y).

If we denote $\psi^\Sigma_\alpha(X,Y) = \psi^\Sigma_{\alpha Y}(X)$, we may look at $\psi^\Sigma_{\alpha Y}(X)$ as a section of the fibration $(\mathbf{A}_{3,1}, S)$, where $\mathbf{A}_{3,1} (\ni X)$ is the base (space-time continuum) and $S = (\oplus_\Sigma S^\Sigma) \otimes \mathbf{R}_{3,1}$ is the fiber, where $\oplus_\Sigma S^\Sigma$ $(\ni \psi^\Sigma_\alpha)$ is the Whitney sum (used in a local theory too) and $\mathbf{R}_{3,1} (\ni Y)$ is the Minkowski space of inner variables Y by which the space-time structure of particles is described (Y are the coordinates in a fiber).

6. Reconstruction of the Space-Time Continuum. Role of Interactions

There is a reason to consider X as the coordinates of the space-time manifold. More exactly, X are the coordinates in the map which grows from the fiber together with the field $\psi^{\Sigma}(X,Y)$. This map is a carrier of the field. In fact, the field $\psi^{\Sigma}(X,Y)$ may evolve alone the diagonal $X = \frac{1}{2}(x + \dot{x})$ in the product of the groups $T_{3,1} \times \dot{T}_{3,1}$ because this field satisfies the Klein – Gordon equation and also some first-order equations. We say that $\psi^{\Sigma}(X,Y)$ may spread by means of a so-called 'horizontal" motion, in result of which the map X is extended. The first-order differential equations follow from the explicit form of the field $\psi^{\Sigma}(X,Y)$ (see [9]). The number of maps is equal to the number of fibers (i.e., fields $\psi^{\Sigma}(X,Y)$). Before the switching on of interactions between the fields $\psi^{\Sigma}(X,Y)$, the maps (and fields) have the independent existence one from another. However, in the result of switching on of interactions (note that their existence is connected with the degeneration of the ground state $\dot{f}_z$ of the bi-Hamiltonian system), another kind of fields (as parameters of the degeneration group) is appeared (compare with [10]). The latter play the role of the connectness coefficients or glue functions. As a result, the maps stick together and the atlas appears. It is the affine space-time $\mathbf{A}_{3,1}$, in which fields may evolve and interact one with another by means of degeneration fields (see [11]).

It is essential that, at the level of states f^{Σ} and $\dot{f}_z$, the charges (quantum numbers Σ connected with the states f^{Σ}) separate from the fields (degeneration parameters are connected with $\dot{f}_z$). The degeneration group is determined as a stationary group of 4-momentum $\dot{p}_\mu$ and is reduced to the unitary transformation of parameters z (in the model $h_8^{(*)}$, it is the $U_i(1)$ group; in the model $h_{16}^{(*)}$, it is the $U_i(2)$ group)[3]. Before transitions $f \to \dot{f}$, there are no states $\dot{f}_z$ yet and, hence, no interactions. We say that, at the level of the bi-Hamiltonian dynamical system, there is the regime of asymptotic freedom. The charges and fields created by them are joined only at the level of the Lagrangian fields $\psi^{\Sigma} \sim \langle \dot{f}_z, f^{\Sigma} \rangle$. Of course, to consider the interaction problem has sense only in the frame of the realistic model $h_{16}^{(*)}$.

[3]There is always a group of diffeomorphisms of $x, \dot{x}$ (or X, Y). With this group, the switching on of the gravitation interaction is connected. There is also a so-called invariant group of states $\dot{f}_z$, with which conservation laws are connected.

It is essential that, along the other diagonal $Y = \frac{1}{2}(x - \dot{x})$, any evolution is impossible. The coordinates Y remain inside the fiber determining the inner structure of particles. We do not loss in generality if to put $Y_\mu = \frac{u_\mu}{k}$, where u_μ are dimensionless coordinates and k is our third fundamental constant. Then the particle field can be written as $\psi^{\Sigma}\left(X, \frac{u}{k}\right)$, and now we may formulate a new correspondence principle in a very simple form: when $k \to \infty$, the bilocal fields $\psi^{\Sigma}(X, \frac{u}{k})$ aspire to the local fields $\psi^{\Sigma}(X) = \psi^{\Sigma}(X,0)$, so that particle field theory becomes local. Having three fundamental constants c, h, k, it is not difficult to get all limit cases and to come to the following important result: when $k \to \infty$ (local limit), $h \to 0$ in order that the product kh become finite (else particle masses become infinite, see (29)). This means that the quantum version of local field theory is impossible. It may be only c-number one.

In [11], by the example of the electromagnetic interaction, it was shown that the quantum version of bilocal fields is free from ultraviolet divergences.

APPENDIX

Here, formula (24) for $F_{\Sigma}(-P^2)$ is derived. In the $h_8^{(*)}$ model, 4-momentum of the bi-Hamiltonian system have form (2). Using the completeness condition for the $\overset{\pm}{\sigma}_\mu$ matrices,

$$\sum_{\mu=1}^{4} (\overset{+}{\sigma}_\mu)^{\dot{\alpha}\beta} (\bar{\sigma}_\mu)_{\gamma\dot{\delta}} = 2\,\delta^{\dot{\alpha}}_{\dot{\delta}}\,\delta^{\beta}_{\gamma},$$

and definitions

$$\hat{N} = \varphi_\alpha \frac{\partial}{\partial \varphi_\alpha} + \bar{\varphi}_\alpha \frac{\partial}{\partial \bar{\varphi}_\alpha}, \quad \hat{Y} = \varphi_\alpha \frac{\partial}{\partial \varphi_\alpha} - \bar{\varphi}_\alpha \frac{\partial}{\partial \bar{\varphi}_\alpha},$$

we have

$$\hat{M}^2 = 2\,\dot{p}_\mu\, p_\mu = -2 \frac{\partial}{\partial \varphi} \bar{\sigma}_\mu \frac{\partial}{\partial \bar{\varphi}} \bar{\varphi} \overset{+}{\sigma}_\mu \varphi = -(\hat{N}+4)^2 + \hat{Y}^2.$$

As skeletons O^{Σ} are the eigenvectors of the operators $\hat{N}$ and $\hat{Y}$: $\hat{N} O^{\Sigma} = N^{\Sigma} O^{\Sigma}, \hat{Y} O^{\Sigma} = Y^{\Sigma} O^{\Sigma}$, where N^{Σ}, Y^{Σ} are their eigenvalues (further, Σ in eigenvalues is omitted) and $w_f = e^{-\beta \bar{\varphi} \varphi}$ (here, $\beta = 1/T_f$), we have $\hat{N}(O^{\Sigma} w_f) = (\hat{N} O^{\Sigma}) w_f + O^{\Sigma} (\hat{N} w_f)$ (and the same for $\hat{Y}$; moreover, $\hat{Y} w_f = 0$). Therefore, on $f^{\Sigma} = w_f O^{\Sigma}$, we have

$$\hat{M}^2 f^{\Sigma} = [-(N+4)^2 + Y^2] f^{\Sigma} -$$

$$- O^{\Sigma} (\hat{N}^2 w_f) - 8 O^{\Sigma} (\hat{N} w_f) - 2 N O^{\Sigma} (\hat{N} w_f).$$

As $\hat{N} w_f = (\varphi \frac{\partial}{\partial \varphi} + \bar{\varphi} \frac{\partial}{\partial \bar{\varphi}}) e^{-\beta \bar{\varphi} \varphi} = 2\beta \frac{\partial}{\partial \beta} w_f$, we may write $\hat{M}^2 f^{\Sigma} = F^0_{\Sigma} f^{\Sigma} - 4\left[\beta^2 \frac{\partial^2}{\partial \beta^2} + (N+5)\beta \frac{\partial}{\partial \beta}\right] f^{\Sigma} = F_{\Sigma} f^{\Sigma}$, where $F^0_{\Sigma} = -(N+4)^2 + Y^2$. Using the identity $\frac{\partial}{\partial \beta} w_f = -\bar{\varphi} \varphi w_f$, we may write $\beta \frac{\partial}{\partial \beta} w_f = -\beta \bar{\varphi} \varphi w_f = \frac{P^2}{4\mu^2} w_f$. The latter equality follows from the Juttner's representation of the Gibbs function $e^{-\beta \bar{\varphi} \varphi} = e^{-(-P^2)/(4\mu^2)}$, where $-P^2 = 2\pi\rho = 4\bar{\varphi}\varphi \bar{z} z$ and $\mu^2 = \bar{z} z/\beta$. Taking this into account, we get

$$F_{\Sigma} = F_{\Sigma}(-P^2) = F^0_{\Sigma} + \frac{N+5}{\mu^2}(-P^2) - \frac{1}{4\mu^4}(-P^2)^2.$$

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